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Intrinsic Shape Analysis: Geodesic Principal Component Analysis for Riemannian Manifolds Modulo Lie Group Actions

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Abstract

A general framework is laid out for principal component analysis (PCA) on quotient spaces that result from an isometric Lie group action on a complete Riemannian manifold. If the quotient is a manifold, geodesics on the quotient can be lifted to horizontal geodesics on the original manifold. Thus, PCA on a manifold quotient can be pulled back to the original manifold. In general, however, the quotient space may no longer carry a manifold structure. Still, horizontal geodesics can be well-defined in the general case. This allows for the concept of generalized geodesics and orthogonal projection on the quotient space as the key ingredients for PCA.

Generalizing a result of Bhattacharya and Patrangenaru (2003), geodesic scores can be defined outside a null set. Building on that, an algorithmic method to perform PCA on quotient spaces based on generalized geodesics is developed. As a typical example where non-manifold quotients appear, this framework is applied to Kendall's shape spaces. In fact, this work has been motivated by an application occurring in forest biometry where the current method of Euclidean linear approximation is unsuitable for performing PCA. This is illustrated by a data example of individual tree stems whose Kendall shapes fall into regions of high curvature of shape space: PCs obtained by Euclidean approximation fail to reflect between-data distances and thus cannot correctly explain data variation. Similarly, for a classical archeological data set with a large spread in shape space, geodesic PCA allows new insights that have not been available under PCA by Euclidean approximation. We conclude by reporting challenges, outlooks, and possible perspectives of intrinsic shape analysis.

This is joint work with Thomas Hotz and Axel Munk also from the University of G¨ottingen.